Evolution and Intelligent Design

Thomas J. Sargent

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David Ricardo, 1816

The introduction of the precious metals for the purposes of money may with truth be considered as one of the most important steps towards the improvement of commerce, and the arts of civilised life; but it is no less true that, with the advancement of knowledge and science, we discover that it would be another improvement to banish them again from the employment to which, during a less enlightened period, they had been so advantageously applied.

Events and ideas

Two sources of ideas that inform central bankers today.

- Experiments and theories before Ricardo.
- Theories and experiments after Ricardo.

Rational expectations

- Equate all subjective distributions.
- Equate subjective distributions to an objective one.

Triumphs of rational expectations theory

- Intelligent design with correct beliefs about consequences of all feasible government policies (including ones that are never observed).
- Rational expectations econometrics.

Questions rational expectations models exclude

- Model misspecification.
- Disputes about consequences of alternative feasible policies.

Evolution and learning

- Do learning models converge to REE?
- Self-confirming equilibria (SCE).
- Learning models that admit misunderstandings about consequences of feasible government policies.
- 800 year process of understanding, abandoning, and seeking monetary anchors.
- Learning inflation-unemployment dynamics in post WWII U.S.

Intelligent design under rational expectations

- Common beliefs.
- Correct views about consequences of policies not chosen.

Intelligent design

$$x_t = \begin{bmatrix} y_t \\ v_t \end{bmatrix}$$

$v_t \sim$ government decisions.

$$x^t = \begin{bmatrix} x_t & x_{t-1} & \dots & x_0 \end{bmatrix}$$

$$f(y^\infty, v^\infty | \rho)$$

$$\rho \in \Omega_\rho$$

- The joint density includes best responses of private agents who choose components of y_t.
- The common beliefs assumption makes parameters describing agents beliefs about endogenous variables disappear from the vector ρ.

Intelligent design

A Pareto criterion that equals expected utility under density $f(x^\infty|\rho)$:

$$\int U(y^\infty, v^\infty|\rho) f(y^\infty, v^\infty|\rho) d(y^\infty, v^\infty). \tag{1}$$

Choose a sequence h of functions

$$v_t = h_t(x^t|\rho), \quad t \geq 0. \tag{2}$$

Intelligent design

Key ingredient: planner has correct ideas about consequences of off-equilibrium path choices.

Common beliefs doctrines

1. Distinction between anticipated and unanticipated policy actions.
2. Designing Ramsey policies.
3. Time inconsistency of time 0 optimal plans.
4. Reputation can substitute for commitment.

Influence of common beliefs doctrines

Because central banks want to implement solutions of Ramsey problems like (2) in contexts like (1) in which the distinction between foreseen and unforeseen policy actions is important, a time inconsistency problem like (3) arises, prompting them to focus on ways like (4) to sustain good expectations.

Why assume common beliefs?

- Just assume it for theoretical and empirical convenience.
- Law of large numbers.

Evolving into equilibrium

- How did the government acquire its model?
- An adaptive least squares learning theory.
- Intelligent design with temporary and possibly misspecified models.
- Transient dynamics.
- Limit points (SCE).

Objects in a self-confirming equilibrium

- True and approximating models $f(y^\infty, v^\infty|\rho)$ and $f(y^\infty, v^\infty|\theta)$.
- Misspecified Ramsey problem

$$\int U(y^\infty, v^\infty) f(y^\infty, v^\infty|\theta) d(y^\infty, v^\infty) \tag{3}$$

- Policy

$$v_t = h_t(x^t|\theta), \quad t \geq 0 \tag{4}$$

Self-confirming equilibrium

Definition

A *self-confirming equilibrium* (SCE) is a parameter vector θ_o for the approximating model that satisfies the data-matching conditions

$$f(y^\infty, v(h|\theta_o)^\infty|\theta_o) = f(y^\infty, v(h|\theta_o)^\infty|\rho). \tag{5}$$

REMARK: It is possible that

$$f(y^\infty, v^\infty|\theta_o) \neq f(y^\infty, v^\infty|\rho) \tag{6}$$

for $v^\infty \neq v(h|\theta_o)^\infty$.

SCE's and learning models

SCE's are limit points of adaptive learning models.

Learning models

- $$\hat{\theta}_{t+1} - \hat{\theta}_t = e_\theta(\hat{\theta}_t, y^t, v^t, t). \tag{7}$$

- $$\hat{v}(h)_t = h_t(x^t|\hat{\theta}_t) \tag{8}$$

  where $h_t(x^t|\theta)$ is the same function (11) that solves the original Ramsey problem under the model $f(y^\infty, v^\infty|\theta)$.

- Deduce restrictions on the estimator e and the densities $f(\cdot|\theta)$ and $f(\cdot|\rho)$ that imply that

  $$\hat{\theta}_t \to \theta_o. \tag{9}$$

REE or SCE?

- If there exists θ_o for which $f(y^\infty, v^\infty|\rho) = f(y^\infty, v^\infty|\theta_o)$ for *all* plans, not just SCE equilibrium ones, then an SCE = REE.
- When $f(y^\infty, v^\infty|\rho) \neq f(y^\infty, v^\infty|\theta_o)$ for some choices of v, convergence to a SCE is the most that can be hoped for.

Why care about SCE-REE gap?

- It doesn't matter to a small agent that its views are incorrect off the equilibrium path.
- It does matter when a government has wrong views off an equilibrium path because they affect its Ramsey plan.

Why use learning models?

- Equilibrium selection.
- Equilibrium modification to improve fits.
  - Asset pricing.
  - Big inflations.
- Good and bad representations of optimal policy rules – Evans and Honkapohja (2003).
- Analyze government policy making with misspecified and disputed models.

Learning to install and uninstall commodity money

- Gold points conceal the quantity theory.
- 800 year transition from full bodied coins throughout the denomination structure, to all coins but one being tokens, to all coins not even being tokens.
- Ricardo (1816).
- Keynes called gold a barbarous relic.

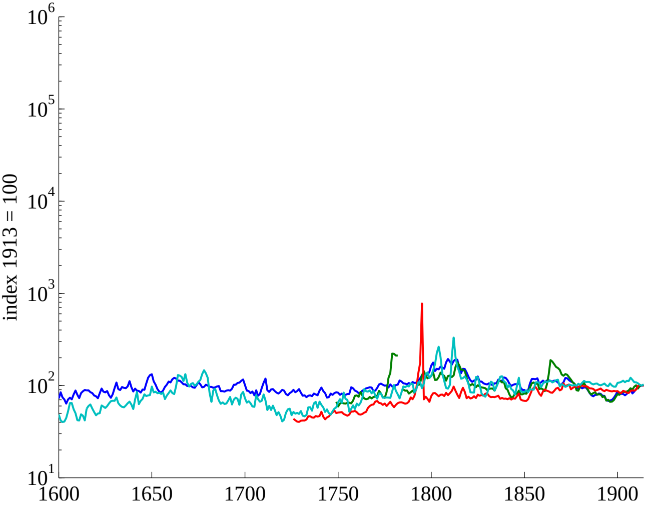
Irving Fisher, 1911

"Irredeemable paper money has almost invariably proved a curse to the country employing it."

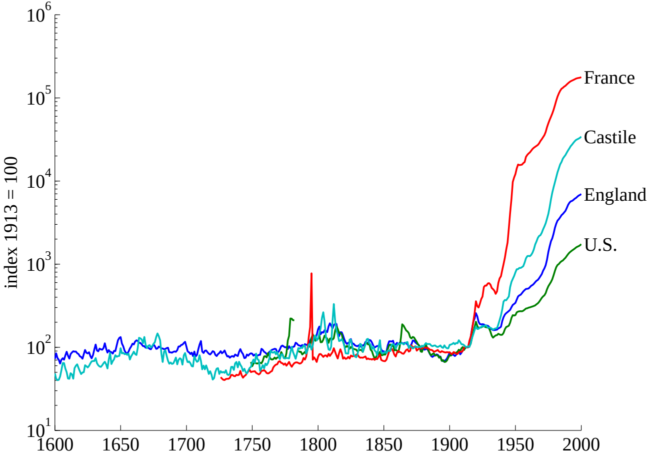
Two threats to price stability

- Political pressure to use printing press to finance government deficits.
- Temptations to exploit inflation-unemployment covariances.

History of prices under barbarous relic



Histories of prices after barbarous relic



Three stories about learning inflation-unemployment dynamics

- The (temporary) conquest of U.S. inflation: $f(x^\infty|\theta) \neq f(x^\infty|\rho)$.
- The Keynesian conquest of U.S. inflation: $f(x^\infty|\theta) = f(x^\infty|\rho)$ but $\hat{\theta}_0 \neq \rho$.
- Evolutions of probabilities that the monetary authority attaches to different submodels of inflation-unemployment dynamics and of the value functions of the submodels.

Three stories

- Story 1: approximating model misses connections among monetary policy, expected inflation, and $U - \pi$ tradeoff.
- Story 2: approximating model correct, except $\hat{\theta}_o \neq \theta_o = \rho$, which leads to wrong off-equilibrium path beliefs about consequences of policy.
- Story 3: learning about three submodels that disagree about consequences of policy choices.

Story 1: Sims's (1988) and Sargent's (1999) (temporary) conquest of U.S. inflation

True (Lucas):

$$U = \rho_0 - \rho_1\rho_3 w_2 + \rho_2 w_1, \tag{10}$$

$$\pi = v + \rho_3 w_2 \tag{11}$$

Approximating (Samuelson-Solow):

$$U = \theta_0 + \theta_1(v + \theta_3\tilde{w}_2) + \theta_2\tilde{w}_1 \tag{12}$$

$$\pi = v + \theta_3\tilde{w}_2, \tag{13}$$

The SCE

Under the approximating model, the government's best policy is

$$v = h(\theta) = \frac{-\theta_1 \theta_0}{1 + \theta_1^2}. \tag{14}$$

There exists a self-confirming equilibrium in which

$$(\theta_0)_o = \rho_0 + \rho_1 h(\theta_o) \tag{15}$$

$$(\theta_1)_o = -\rho_1. \tag{16}$$

Reason for suboptimality of SCE

- The data-matching restriction (15) pinpoints how the government mistakenly ignores the effect of its policy choice v, which equals the public's expected rate of inflation, on the level of the Phillips curve.
- population regression coefficient of U on π is

$$\theta_1 = \frac{-\rho_1 \rho_3^2}{\mathrm{var(v)} + \rho_3^2}.$$

- If v were generated randomly with enough variance, then even though it fits the wrong model, the government would estimate a Phillips curve slope θ_1 of approximately zero and would according to (14) set v approximately to its optimal value of 0 under the true model.
- But within an SCE, v doesn't vary enough for the government to estimate a θ_1 close enough to zero for that to happen.

Furthermore, the outcome that $\hat{\theta}_t \to \theta_o$ means that the variation of v_t that occurs along transient paths is insufficient to allow the government's model to approximate the data in a way that tells it to implement what would be the optimal policy under the true model.

Escaping the SCE

- That is not the end of the story.
- Shocks and the adaptive model's endogenous stochastic dynamics occasionally make v vary enough for the government to discover a (too strong) version of the natural rate hypothesis, too strong because it mistakenly asserts that there is no tradeoff whatsoever between π and U.

Escapes from the SCE

- Fate consigns the economy to experience recurrent episodes in which ‘a most likely unlikely’ sequence of w’s lowers the unconditional correlation between U and π, which in turn prompts the government’s estimates $\hat{\theta}_t$ to induce the government to push v_t downward from its self-confirming value.
- This process generates data that weakens the unconditional correlation between inflation and unemployment and drives v even lower.
- The ultimate destination of this ‘escape’ from a self-confirming equilibrium is that the government estimates that θ_1 is 0, prompting it to set v_t at the optimal value 0.

Messages of the misspecified model parable

- Benefits and costs of overfitting to recent data.
- Different reason for suboptimality than time-inconsistency.
- A lack-of-experimentation trap.
- Danger that inflation stabilization is temporary because model has been fit to transient data.

Story 2 – Primiceri (2005)

- $f(x^\infty|\rho) = f(x^\infty|\theta_o) \neq f(x^\infty|\hat{\theta}_0)$.
- f is Solow-Tobin model.
- Calibrate $\hat{\theta}_0$ from 1960's data.
- $\hat{\theta}_0$ underestimates persistence of inflation relative to θ_o in SCE=REE.

Evolution of beliefs in Primiceri's story

Determinants of policy variable v_t in Primiceri model: time t estimates of:

- Natural rate of unemployment.
- Persistence of inflation.
- Slope of Phillips curve in King and Watson's Keynesian direction.

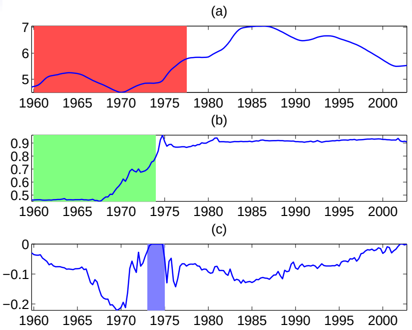


Figure: Evolution of policy-maker's beliefs about: (a) the natural rate of unemployment; (b) the persistence of inflation in the Phillips curve; and (c) the slope of the Phillips curve in King and Watson's (1994) Keynesian direction. Shaded areas are where the government (a) underestimates the natural rate of unemployment, (b) underestimates the persistence of inflation, and (c) thinks that the sacrifice ratio is very large. Source: I have adapted a figure of Primiceri (2006).

Evolution of beliefs in Primiceri's story

- Primiceri calibrates initial condition $\hat{\theta}_0$ that sends economy on path in which transient movements in $\hat{\theta}_t$ produce a path of U, π that fits emergence of big inflation in the 70s and stabilization the 80s and 90s.
- v_t looks like a short term real interest rate.
- $\hat{\theta}_t \rightarrow \theta_o = \rho$.

Primiceri's story

- Optimistic conclusion.
- Caveat: interpretation relies on direction of fit (King and Watson (1994)).

Interpreting the diminished U, π correlation

- Primiceri (2006) attributes the inflation of the 1970s to the high perceived sacrifice ratio that Keynesian Phillips curve models presented to policy makers.
- He assumes that the Fed relied exclusively on a version of the Solow-Tobin model and is silent about why the Fed disregarded the recommendations of the Lucas (1972, 1973) model.
- Our third story focuses on how sacrifice ratios differ so much across submodels.
- Different models interpret the diminished, near-zero contemporaneous covariance between inflation and unemployment that had emerged by the mid 1970s very differently.
- That matters.

Interpreting the diminished U, π correlation

- In a Keynesian Phillips curve, this diminished covariance flattens the short-term tradeoff, making the authorities believe that a long spell of high unemployment would be needed to bring inflation down, prompting Keynesian modelers to be less inclined to disinflate.
- But for a classical Phillips curve, the shift toward a zero covariance steepens the short-term tradeoff, making the authorities believe that inflation could be reduced at less cost in terms of higher unemployment.
- Thus, a classically-oriented policy maker would have been more inclined to disinflate.

Story 3: the evolution of model mixing

- Cogley and Sargent (2005)
- $f(x^\infty|\theta)$ mixes three models, where θ includes model mixing probabilities.
- Update model weights by Bayes' law.
- Three submodels:
  - Solow-Samuelson (SS): an exploitable trade-off.
  - Solow-Tobin (ST): exploitable trade-off except in "long run".
  - Lucas (L): no exploitable trade-off.

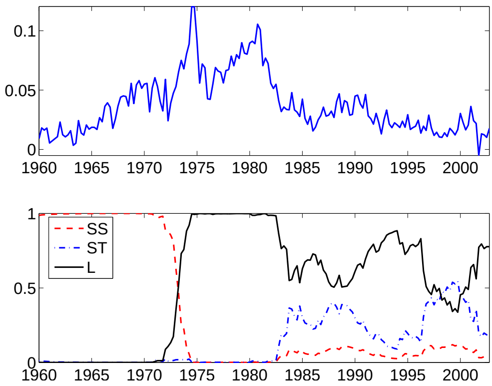


Figure: Top panel: CPI inflation. Bottom panel: Bayesian posterior model weights on the Samuelson-Solow (SS), Solow-Tobin (ST), and Lucas (L) models.

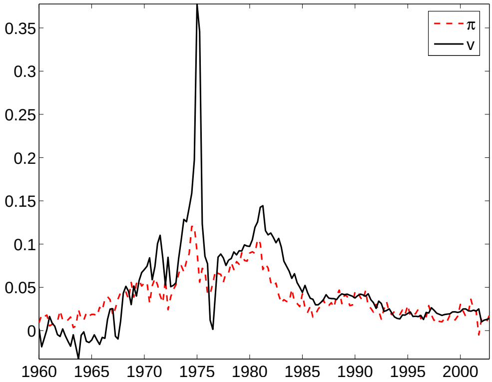


Figure: CPI inflation and recommendation from Phelps problem.

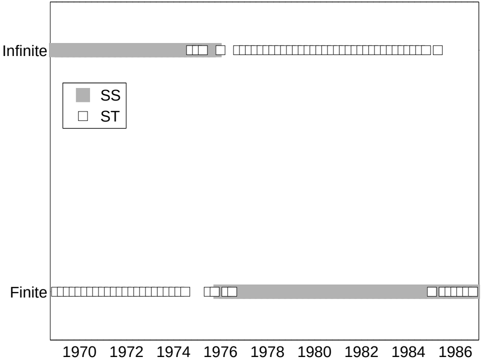


Figure: Losses under no-inflation policies for Samuleson-Solow (SS) model and Solow-Tobin (ST) models.

Lessons

- Low probability models remain influential when their value functions are very low under policies recommended by high probability models.
- Symmetrically, high probability models lose influence when they imply moderate losses under policies recommended by low probability models.
- An endogenous ‘worst-case’ analysis.
- SS and ST models had to give permission to stabilize – that delayed the stabilization.

Lessons

- Drive a stake through misspecified models.
- Who is authorized and knows enough to do that?

Ultimate lesson

- Explanations for U, π paths in all three models would break down if we had given the monetary authority the one-period loss function π^2 instead of $(U^2 + \pi^2)$.
- This occurs despite the different kinds of misspecification featured in our three stories.
- A defense for inflation targeting based on fear of model misspecification.

Inflation targeting

When we asked the Fed for more, we usually got less.

Concluding remarks

I admire the quote from Ricardo. It conveys respect for the struggles of our predecessors and the monetary institutions that they created, and confidence that, armed with new models and technologies, we can do better.